Electrical behavior of memristive networks

Research background

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February 10, 2015
1. Summarized history of the memristor
   - Timeline
   - Leon Chua’s memristor

2. Mathematical models of memristor
   - Memristive dynamical systems
   - HP TiO$_2$ memristor
   - I-V curves

3. Analytic solutions of memristor dynamics
   - Bernoulli-type memristor
   - Analytic solutions
   - Bad news

4. Memristors that exist and compute
   - Mechanics of memristance
   - Morphology
   - Computing applications

5. Potential directions for future work
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A bit of history

Timeline
Leon Chua’s memristor

- Resistive Switching
- Memristor Theory
- Silicon Slowdown
- Analytic Results

200 yr
Leon Ong Chua (1971) realized that not all circuit variables were related. He postulated a new passive circuit element which instantaneous resistance depends on the history of the input. The **memristor** is a nonlinear resistor with memory.

*D. B. Strukov et al. (2008). The missing memristor found. Nature, 453(7191), 8083. doi:10.1038/nature06932*
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Memristive networks
Memristor and memristive systems

**Memristor** \( (V = 0 \Leftrightarrow I = 0) \)

\[
V = R(w)I \quad (1)
\]

\[
\dot{w} = f(I) \quad (2)
\]

**Memristive system**

\[
V = R(w, I)I \quad (3)
\]

\[
\dot{w} = f(w, I) \quad (4)
\]
Figure 1: HP TiO$_2$ memristor

\[ V(t) = R_{\text{off}} \left( 1 - \frac{\mu}{D^2} R_{\text{on}} q(t) \right) I(t) \]

\[ \mathcal{M}(q) \]

(5)
HP memristor response

Figure 2: Applied voltage (blue) and resulting current (green) as a function of time. The numbers 1-6 label successive waves in the applied

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Jakob Bernoulli’s equation

Charge controlled memristor, voltage driven

\[ I(t) = \mathcal{M}^{-1}(q) V(t) \quad (6) \]

\[ \dot{I}(t) - \frac{\dot{V}(t)}{V(t)} I(t) = -\frac{d\mathcal{M}}{dq} \frac{1}{V(t)} I^3(t) \quad (7) \]

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Bernoulli’s equation

\[ \dot{y} + p(t)y = s(t)y^n \quad (8) \]

\[ y(t)^{1-n} = \frac{1}{m(t)} \left[ B + (1 - n) \int_{t_0}^{t} m(\tau)s(\tau)d\tau \right] \quad (9) \]

\[ m(t) = \exp \left( (1 - n) \int_{t_0}^{t} p(x)dx \right) \quad (10) \]

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Memristive networks
Analitic solutions

Charge controlled memristor, voltage driven

\[ I(t) = \mathcal{M}^{-1}(q)V(t) \]

\[ \dot{I}(t) - \frac{\dot{V}(t)}{V(t)} I(t) = -\frac{d\mathcal{M}}{dq} \frac{1}{V(t)} I^3(t) \]

Has the analitic solution

\[ I(t) = V(t) \left[ \mathcal{M}_0^2 + 2 \int_0^t \frac{d\mathcal{M}(q)}{dq} V(\tau) d\tau \right]^{-\frac{1}{2}} \]

Using memristance from Eq. (5) we get all the responses observed.
### I-V memristor curves

<table>
<thead>
<tr>
<th>$\alpha = 1$</th>
<th>$\beta_1$</th>
<th></th>
<th>$\beta_1$</th>
<th></th>
<th>$\beta_1$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{W}(t) = k_1 V(t)$</td>
<td>-0.499</td>
<td><img src="image1" alt="Graph" /></td>
<td>-0.1</td>
<td><img src="image2" alt="Graph" /></td>
<td>+0.5</td>
<td><img src="image3" alt="Graph" /></td>
</tr>
<tr>
<td>$\dot{W}(t) = k_1 V^2(t)$</td>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
<td><img src="image7" alt="Graph" /></td>
<td><img src="image8" alt="Graph" /></td>
<td><img src="image9" alt="Graph" /></td>
<td><img src="image10" alt="Graph" /></td>
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<table>
<thead>
<tr>
<th>$\alpha = 2$</th>
<th></th>
<th>$\beta_1$</th>
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<th></th>
<th>$\beta_1$</th>
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<tbody>
<tr>
<td>$\dot{W}(t) = k_1 V(t)$</td>
<td><img src="image11" alt="Graph" /></td>
<td><img src="image12" alt="Graph" /></td>
<td><img src="image13" alt="Graph" /></td>
<td><img src="image14" alt="Graph" /></td>
<td><img src="image15" alt="Graph" /></td>
<td><img src="image16" alt="Graph" /></td>
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</tr>
<tr>
<td>$\dot{W}(t) = k_1 V^2(t)$</td>
<td><img src="image17" alt="Graph" /></td>
<td><img src="image18" alt="Graph" /></td>
<td><img src="image19" alt="Graph" /></td>
<td><img src="image20" alt="Graph" /></td>
<td><img src="image21" alt="Graph" /></td>
<td><img src="image22" alt="Graph" /></td>
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</table>
Bad news
The physics of the models is inaccurate

The current theoretical models of memristors are criticized:
- Doped region diffuses to equilibrate concentration gradient. Memristor forgets.
- Memristor’s internal state cannot be continuous: Landauer’s principle of the minimum energy costs for information processing.

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5. Potential directions for future work
Ag-H$_2$O-Pt based atomic switches: morphology
Macroscopic self-assembly: Pelesko chain

J.P. Carbajal  Memristive networks
"... when confronted with a new device, one needs to determine whether it has a natural basis for computation that is different from familiar paradigms."
MEMRISTOR CELLULAR AUTOMATA
AND MEMRISTOR DISCRETE-TIME
CELLULAR NEURAL NETWORKS

MAKOTO ITOH
Department of Information and Communication Engineering,
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Fukuoka 811-0295, Japan

LEON O. CHUA
Department of Electrical Engineering and Computer Sciences,
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There are several paths to pursue in the short time scale. All approaches are simulation based with potential collaboration with experimental groups.

### Low level

<table>
<thead>
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<th>Statistical properties</th>
</tr>
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<tbody>
<tr>
<td>- Aim: device variability.</td>
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<tr>
<td>- Simulated growth.</td>
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<tr>
<td>- Properties of morphology.</td>
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</tbody>
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### High level

<table>
<thead>
<tr>
<th>Computation</th>
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<tr>
<td>- Aim: models of computation.</td>
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<tr>
<td>- Simplify/idealized models.</td>
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<tr>
<td>- Large networks.</td>
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</tbody>
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Thank you!